

classix Software GmbH

Fundamentals of inventory sampling

**A description of the mathematical-statistical methods of the
inventory sampling with classix**

classix

a class of its own

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1 Mathematical basics of inventory sampling

Once we know a warehouse's mean μ , we can calculate the total value of the warehouse Y by multiplying the mean with the total number of parts N . For this purpose the warehouse's mean μ is approximated by the mean \bar{y} of the sample \hat{y} of n inventory positions. The mathematical context then derives from the law of large numbers:

$$\lim_{k \rightarrow \infty} \frac{\sum_{i=1}^k \bar{y}_i}{k} = \mu$$

Alternatively a supported extrapolation can be done and thereby derive μ from \bar{y} .

1.1 Requirements to inventory sampling

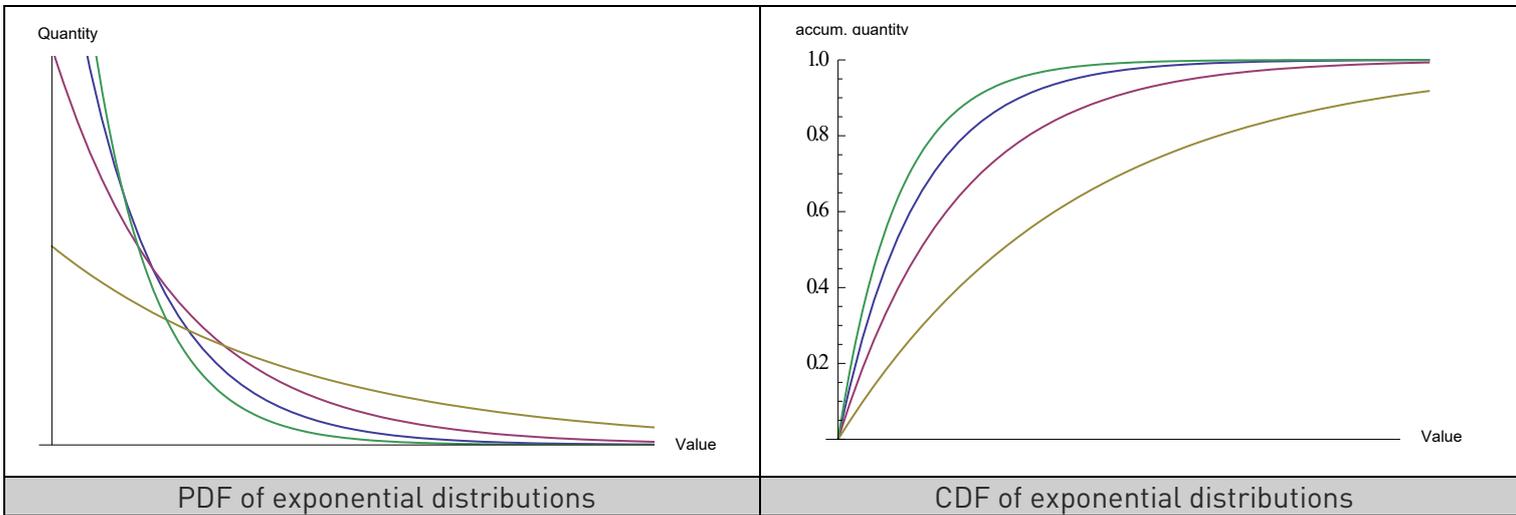
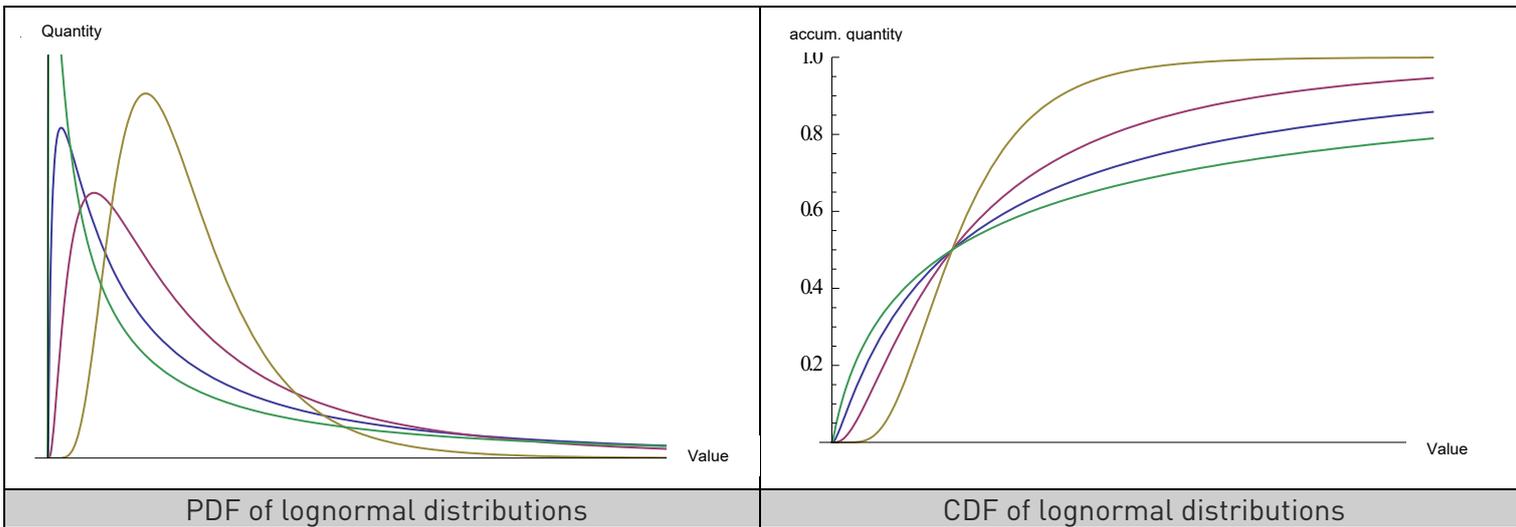
According to HGB § 241 Section 1, "approved mathematical-statistical methods on the basis of sampling" may be used, if the "quality of an inventory set up in this way equals the quality of a physical inventory". According to (IDW, 1981, S. 13), (AWV, 1978, S. 21), this means that with a probability of 95% the relative error (e_r) of an inventory sampling must not be greater than 1%.

Assuming \bar{y} is normally distributed, one can conclude from tables of standard normal distributions that the mean \bar{y} will be with a probability of 95% in the interval $[-1.96\sigma, 1.96\sigma]$ (σ defines the standard deviation) or in other words the confidence value ("t-value") is $t = 1.96$. If an error of 1% is accepted, then one has to achieve for \bar{y} , that 1.96σ is smaller than 1% of the total value of the warehouse.

The assumption of a normal distribution is justified for large enough sample sizes (AWV, 1978, S. 12), (IDW, 1981, S. 68). For a required statistical confidence of 95.5 %, a "t-value" of $t = 2$ is needed according to AMV (AWV, 1978, S. 20). For a statistical confidence of 95% this corresponds to the above mentioned value of $t = 1.96$ under the assumption of a normal distribution.

1.2 Distribution of parts in warehouses

In general one assumes that the number of positions in warehouses is exponentially or lognormal distributed related to the positions' values. For distributions, there are two frequently used functions in statistics: The **probability density function** (PDF) and the **cumulative distribution function** (CDF). The former determines how frequent a value occurs. For the CDF the values' frequencies have been accumulated.



To describe a warehouse's distribution, one creates a histogram for the PDF, plotting the parts' values against the total number. Accumulating the number gives the CDF. Subsequently accumulating the values, one receives the so-called Lorenz curve.

1.3 Complete inventory in the border area

To remove particularly high and low values from the inventory sampling and thereby reduce the risk of the inventory, a complete inventory is done in the border area. On the one hand this includes all positions without quantity expected. On the other hand this includes the 5% most valuable parts, which in most warehouses make up for 50% of the warehouse's total value. Hereby we meet the requirement defined in (IDW, 1981, S. 69):

„Due to the “warehouse phenomenon” it is sufficient to carry out a complete inventory for a fraction of 3 to 5% of all positions in a warehouse, to cover 45 to 50% of the warehouse's total value.”

1.4 Overview over the methods

The remaining part of the warehouse can be tested by different methods. The different methods define how samplings are drawn (**stratified or unlimited random sampling**) and how the

results are extrapolated (**free or supported extrapolation**). Additionally one can use sequential tests, which do not determine the total value of the warehouse but just check whether the error amount is acceptable.

Sampling	Extrapolation
stratified	free
unlimited	Regression estimation
	Difference estimation

The choice of the method for the sampling is in principle independently of the method for the extrapolation. However, according to Angele (Angele, 1989, S. 276) a combination of a stratified random sampling and a supported extrapolation is not favourable.

For the stratified random sampling first the warehouse is decomposed into strata. Afterwards the number of samples from every stratum is defined to avoid a concentration of samples. The unlimited random sampling does not define such preconditions.

Depending on the type of extrapolation, different estimated values of the inventory value are determined from the sampling (and accordingly different sample variances). Afterwards the warehouse value is determined by multiplication of the estimated mean and the number of items: $Y \approx N \cdot \mu'$

Procedure	Estimated mean μ' for μ	Sample variance $v = s_h^2$
Free extrapolation	\bar{y}	$\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1}$
Regression estimation	$\bar{y}_{tr} = \bar{y} + b \cdot (\bar{X} - \bar{x})$	$\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1} \cdot (1 - R^2)$
Difference estimation	$\bar{y}_{tr} = \bar{X} + \bar{y} - \bar{x}$	$\frac{\sum_{i=1}^n (x_i - \bar{x})^2 + \sum_{i=1}^n (y_i - \bar{y})^2 - 2 \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$

If one requires $N \cdot \mu'$ to be correct to a probability with confidence t except for an error of d , it follows that $V(N \cdot \mu') \leq \left(\frac{d}{t}\right)^2$ needs to be valid (Cochran, 1972, S. 129). From this follows that one needs to draw $n \geq \frac{t^2 N^2 v}{d^2 + t^2 N v}$ samples (v defines the sample variance of the different procedures respectively). The sample variance needs to be approximated by known values, as y_i and \bar{y} are unknown prior to drawing the samples. Usually y is approximated by an auxiliary value x , the **coefficient of determination** R^2 and the sample variance of the difference estimation through former inventory data and the sample size n through the population size N .

In order to reduce the number of samples whilst ensuring having the same accuracy or in other words increasing the accuracy with the same number of samples, it is tried to reduce the variance or the standard deviation respectively.

1.5 Random selection (drawing of the samples)

In IDW (IDW, 1981, S. 65) the requirements for drawing a sample are as follows:

“Precondition for the application of all mathematical sampling procedures is, that all elements of a sample are selected randomly from the individual elements (stock items) of a carefully limited basic population (collective of a warehouse) [...]. This means, that

- for a so called non stratified random selection every stock item needs to have the same from zero differing chance to be part of the random selection or
- for a so called stratified random selection every stock item needs to have a calculable from zero differing chance to be part of the random selection

As a selection procedure lottery, the procedure of random numbers, the systematically selection with random start point, the end number procedure etc. are possible.”

Thus, there is the free random selection and the stratified random selection.

1.5.1 Stratified random Sampling

One possibility to reduce the variance is to divide the warehouse L into strata depending on the article value.

The stratified random sampling contrasts the free random sampling and is therefore called supported random sampling.

Then from each of the L strata a sample \hat{y}_h is drawn. Subsequently a mean for the whole warehouse is calculated from the means of the samples from the strata, taking the strata's sizes N_h into account:

$$\bar{y}_{st} = \frac{\sum_{h=1}^L \bar{y}_h N_h}{N}$$

For well-chosen strata, [Cochran, 1972, S. 163] stated the following correlation:

$$V(\bar{y}_{st}) \approx \frac{V(\bar{y})}{L^2}$$

This justifies why stratification is done for inventory sampling.

1.5.1.1 Define stratum borders

There are different approaches how to build the strata. All of them try to minimize the variance $V(\bar{y}_{st})$.

1.5.1.1.1 Method 1 (Dalenius & Hodges)

[Cochran, 1972, S. 159f]

This method was proposed by [Dalenius & Hodges, 1959]. First the range of values is divided in equal classes. (e.g.: 0%-5%, 5%-10% etc. of the maximum value)

For each class k we determine how many parts N_k are contained in this class. From this number the square root $\sqrt{N_k}$ is taken.

Then the strata are set up, in such a way that the sums of roots for all strata are roughly equal. To do so it is recommended to accumulate the roots in a table and divide the total sum by the number of strata L .

If the sums of the strata differ too much, a refinement of the classes might help (e.g. from 5%-steps to 2.5%-steps).

1.5.1.1.2 Method 2 (Ekman)

(Cochran, 1972, S. 161)

For the second method, which was proposed by (Ekman, 1959), the strata are chosen in such a way that for upper and lower limit of the h -th stratum x_h and x_{h-1} respectively the product $N_h(x_h - x_{h-1})$ is constant for different h s.

1.5.1.1.3 Method 3 (GESTIN)

(GESTIN – Wartungsdokumentation V4 Revision 6, S. 41f)

The third method is used by GESTIN to determine strata. First the upper limit of the first stratum is defined manually. Afterwards the above strata are defined by the condition, that the sample variance s_i^2 of the i -th stratum multiplied with the size of the stratum squared N_i^2 does not exceed the value $\left(\frac{5}{4}N_1\right)^2 \cdot s_1^2$ with respect to the first stratum.

An empty stratum first shows a size of N and a sample variance s^2 of zero. Now, the respective stratum limit is continuously increased until a further increase would result in $N_i^2 \cdot s_i^2 > \left(\frac{5}{4}N_1\right)^2 \cdot s_1^2$. This value is used as an upper limit.

As an abort criterion an upper limit defined by the user is used. It is needed to make sure that the legal requirements for a complete enumeration are met.

1.5.1.2 Number of individual samples

Once the strata are set, one can determine, how many individual samples needs to be drawn from each stratum to achieve the desired precision.

To do so, one first calculates the total number of individual samples and then divide these into the strata. To meet the requirements of inventory sampling, first the desired variance has to be calculated.

Given a stratification and may y_{hi} be the i -th value of the sample \hat{y}_h of the value n_h of the stratum h , then the standard deviation s_h is determined by $s_h^2 = \frac{\sum_{i=1}^{n_h} (y_{hi} - \bar{y}_h)^2}{n_h - 1}$ or by the calculation rule given by the extrapolation (see the table of the different procedures). The total number of individual samples than amounts to (Cochran, 1972, S. 130):

$$n = \frac{(\sum_{h=1}^L N_h s_h)^2}{V(\bar{y}_{st}) + \sum_{h=1}^L N_h s_h^2} \geq \frac{t^2 \cdot (\sum_{h=1}^L N_h s_h)^2}{d^2 + t^2 \cdot \sum_{h=1}^L N_h s_h^2}$$

as $V(\bar{y}_{st}) \leq \left(\frac{d}{t}\right)^2$.

(IDW, 1981, S. 68) gives 100 as a minimum for the number of individual samples: "For mean-value estimation the size of a sample in the case of stratified extrapolation for example should not be below 100 individual samples."

Furthermore, in (AWV, 1978, S. 20) a minimum of 2% of the total number N is announced for the number of individual samples n : “For the size of a sample n [...] in general, out of reasons of confidence, the principle holds: Minimum of n of N : 2% of all positions.”

According to (Neyman, 1934) the ideal number of samples for each stratum then is

$$n_h = n \cdot \frac{N_h s_h}{\sum_{h=1}^L N_h s_h}$$

These are now randomly drawn within each stratum.

1.5.1.3 Number of strata

The first both methods for determining the borders of the strata rely on the fact that the number of strata L is given. To achieve this, (Kabuss, 1991) provides an iterative method in section 2.4.4.1. From “years of experience” Kabuss assumes that it is ideal to have the sample size per stratum n_h to be between 12 and 14 (Kabuss, 1991, S. 34).

First, an initial value for L is chosen. With the help of this value the strata’s boundaries are determined by one of the former methods (Dalenius & Hodges or Ekman). Afterwards one calculates the values for n and for each stratum n_h . Then the average of the n_h s is calculated. If it is smaller than 12, L is decreased, if it is greater than 14, L is increased. Afterwards this process is repeated with this new L as initial value. Is the average of the n_h s in between 12 and 14, the stratification is valid.

An optimum range is useful, because on the one hand the sample size needs to be reduced as far as possible, in order to reduce the variance and on the other hand the stratum cannot be too small, in order to get a sample result \bar{y} , which is approximately normally distributed. This is very important, to be able to guaranty the accuracy of the statement of the confidence level and the fault tolerance.

1.6 Extrapolation

After determining the sample size for the different strata, the actual inventory of the sample elements is checked. In order to get the mean of the sample (\bar{y}), the sum of all values is divided by the size n : $\bar{y} = \frac{\sum_{i=1}^n y_i}{n}$. This value can be used directly to approximate μ . Alternatively one can use an estimation procedure, in order to refine the approximation.

1.6.1 Regression estimation

If the counted values of the items are strongly related to the auxiliary values (typically the book values or the values of a former inventory), which are known, regression estimation can be performed: “On the one hand often an inventory accounting, with a record about every stock value is available. On the other hand it is obvious to include values of former periods if the warehouse structure is constant (presumptive values).” (Angele, 1989, S. 106)

The regression estimation is a generalisation of the difference estimation and is part of the supported estimation.

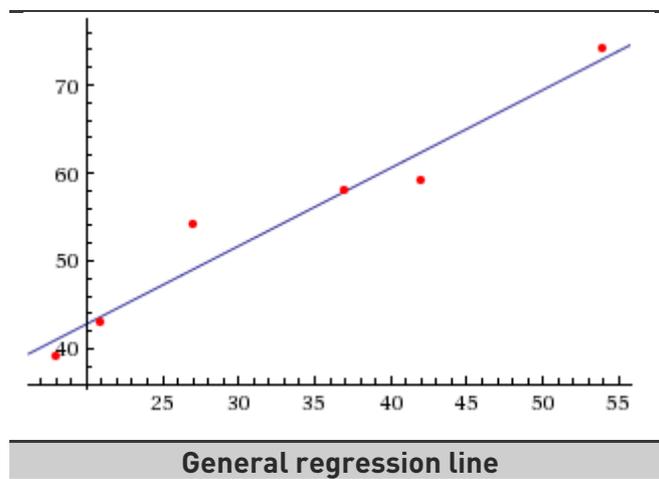
1.6.1.1 Coefficient of determination

It is assumed that there is a linear relationship between the counted values and the auxiliary values. The coefficient of determination

$$R^2 = \frac{(\sum_{i=1}^N (y_i - \bar{y})(x_i - \bar{x}))^2}{\sum_{i=1}^N (x_i - \bar{x})^2 \cdot \sum_{i=1}^N (y_i - \bar{y})^2}$$

however needs to be estimated at the beginning in order to determine how well the assumption is met. For this purpose one uses existing inventory data or alternatively draws a sample. The coefficient of determination is used to determine the sample size necessary to be able to make a sufficient reliable statement.

To increase the security, this value is sometimes reduced. Attention should be paid to the fact that a reduction of R^2 leads to enormous increase in sample size. [Angele, 1989, S. 113]



1.6.1.2 Estimation

First a sample is drawn out of the warehouse $((x_1, y_1), \dots, (x_n, y_n))$, in which x_i is the auxiliary value and y_i is the actual value of the i -th item. It is shown later, how n is determined.

Let Y be the total value of the warehouse, \bar{y} the mean of y_i , X the sum of the auxiliary values and \bar{x} the mean of x_i . So the approximated mean \bar{y}_{lr} by linear regression is given by [Angele, 1989, S. 106], [Cochran, 1972, S. 226]:

$$\bar{y}_{lr} = \bar{y} + b \cdot (\bar{X} - \bar{x})$$

The total value of the warehouse can now be approximated by the multiplication of the approximated mean with the total number of items in the warehouse N ($Y \approx N \cdot \bar{y}_{lr}$).

1.6.1.3 Determination of the parameter

As X and N are known, it is possible to calculate $\bar{X} = \frac{X}{N}$. b defines the convergence of counted value and auxiliary value B to the slope of the regression line. Choosing $b = 1$, one gets the difference estimation. b can be determined (analogously to the coefficient of determination R^2) by means of counted value and auxiliary value of a former inventory or by drawing a sample at the beginning according to the following equation ($Cov(x, y)$ is the covariance of x and y):

$$b = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{Cov(x, y)}{V(x)^2} \approx B$$

1.6.1.4 Determination of the sample size

According to Angele (Angele, 1989, S. 107) the variance of the approximation can be calculated by:

$$V(N \cdot \bar{y}_{lr}) = N^2 \cdot \frac{\left(1 - \frac{n}{N}\right)}{n} \cdot \sum_{i=1}^n (y_i - \bar{y})^2 \cdot (1 - R^2)$$

To get the absolute error d with a confidence with coefficient t , $V(N \cdot \bar{y}_{lr}) \leq \left(\frac{d}{t}\right)^2$ needs to be true [t can be read out of tables for the [normal distribution](#) and has a value of 1.96 for a probability of 95 %.]

Let $v = \sum_{i=1}^n (y_i - \bar{y})^2 \cdot (1 - R^2)$ be the sample variance, then the requirement can be written as:

$$vN^2 \cdot \frac{\left(1 - \frac{n}{N}\right)}{n} \leq \left(\frac{d}{t}\right)^2 \Rightarrow n \geq \frac{vN^2}{vN + \left(\frac{d}{t}\right)^2}$$

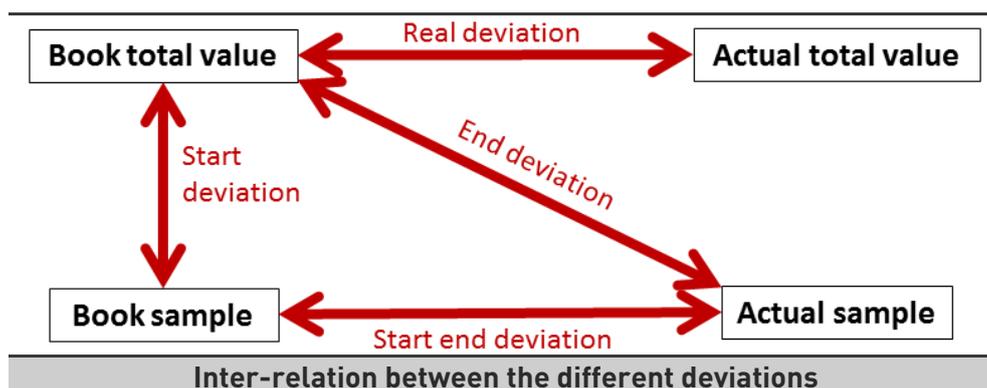
Unfortunately, the last term cannot be calculated directly, as the values of n and y_i are needed to calculate v . Because of that Angele (Angele, 1989, S. 139) specifies the following estimation:

$$n \geq \frac{vN^2}{vN + \left(\frac{d}{t}\right)^2} \approx \frac{t^2 \cdot N \cdot \sum_{i=1}^N (x_i - \bar{x})^2 \cdot (1 - R^2)}{d^2 \cdot N + t^2 \cdot \sum_{i=1}^N (x_i - \bar{x})^2 \cdot (1 - R^2)}$$

1.7 Deviation

If the positions of the sample are determined, the total value can be extrapolated from these positions. This extrapolation differs obviously from the real book total value. This difference relatively to the book total value is per construction to a probability of 95% smaller than 1% and is defined as start deviation.

Actually interesting for the inventory is the real deviation between the book total value and the actual total value. This deviation can only be determined through a complete inventory. Because of that, it is instead checked that the end deviation is not too large. The end deviation consists additively from the start deviation and the start end deviation.



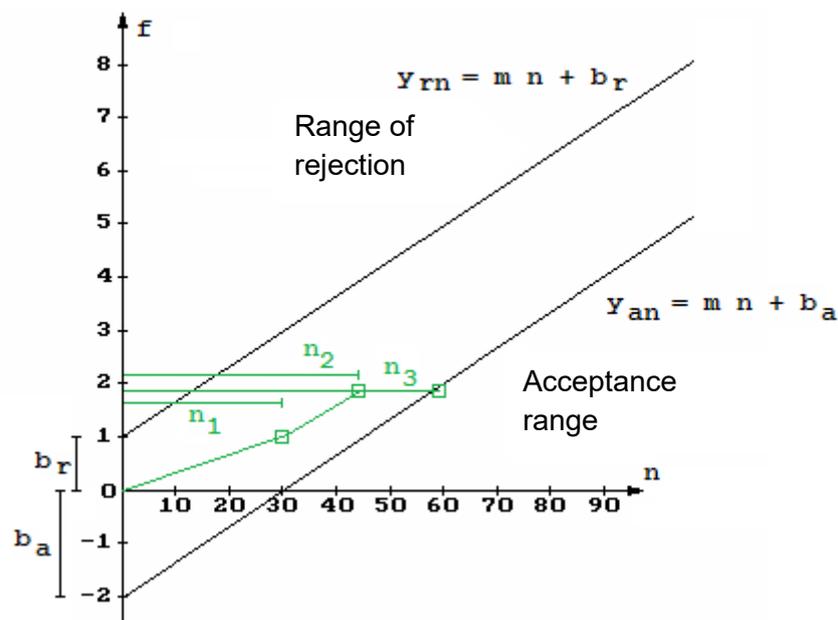
1.8 Sequential test for automatical warehouses

Sequential testes are advisable for warehouses with a very high accuracy. This needs to be kept in mind as the effort is theoretically unlimited. Therefore the effort might be higher than the effort for a complete inventory in some situations.

First the amount of items, which are not moved during the inventory period, needs to be accounted physically. However, it is possible to use sampling procedures. (AWV, 1980, S. 1f)

1.8.1 Basic procedure

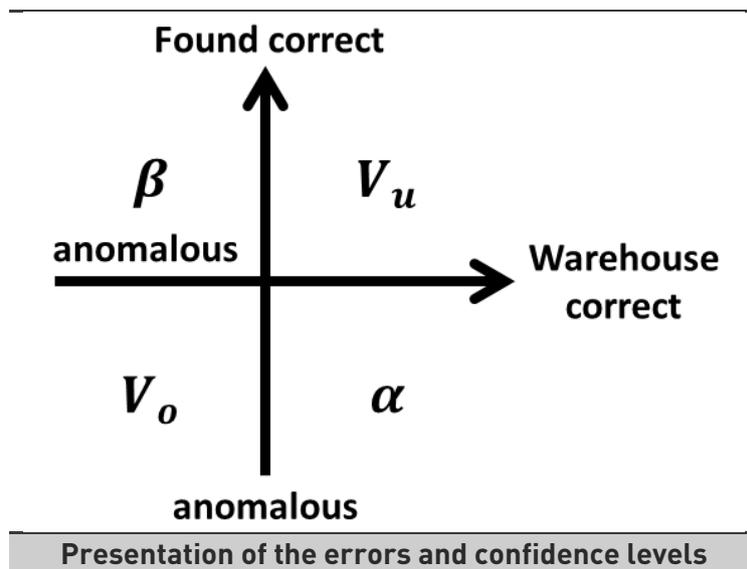
The theoretical basics of sequential tests trace back to Wald (Wald, 1947). First the parameter m , b_a and b_r are determined. Afterwards a sample of size n_1 is drawn from the warehouse and it is checked how often book inventory and actual inventory are the same. If this amount is smaller or equal to y_{an} , the inventory will be counted as accepted. If the amount is smaller or equal to y_{rn} , the inventory will be rejected. For all other values a sample is drawn, resulting in an amount of n_2 drawn items. Afterwards the procedure is repeated until the result is fixed. In order to avoid an exorbitant effort, a maximum of drawing should be defined at the beginning. A possible choice is $n \leq 0,05 \cdot N$.



Schematical presentation of the procedure of a sequential test

1.8.2 Determination of the parameter

First the parameters for the probability need to be determined. Here p_u is the lower and p_o the upper limit for the error portion (AWV, 1980, S. 6). Furthermore, α defines the risk, having a rejected inventory for a correct warehouse and β the risk of having an accepted inventory for an anomalous warehouse (AWV, 1980, S. 7). Odenthal (Odenthal, S. 28) calls these confidence levels. Thereby $V_o = 1 - \beta$ defines the confidence, that an anomalous warehouse is rejected and $V_u = 1 - \alpha$ the confidence that a correct warehouse is accepted.



The lower the values for the parameters p_u , p_o , α and β , the stricter the testing (AWV, 1980, S. 16f). Obviously $p_u \leq p_o$ needs to be true.

The AMV intends values of p_u : 1 – 2% and p_o : 3 – 10% (AWV, 1980, S. 16). In (AWV, 1985, S. 21) these are specified as p_u : 0,5% and p_o : 1%. (AWV, 1980, S. 17) furthermore defines the vales of α and β to be α : 5% and β : 5%. Please remember for the following equations that 1% = 0,01.

If the parameters are set, the following can be calculated directly (AWV, 1980, S. 11), (AWV, 1985, S. 15):

$$m = \frac{\log(1 - p_u) - \log(1 - p_o)}{\log(p_o(1 - p_u)) - \log(p_u(1 - p_o))}$$

$$b_a = \frac{\log(\beta) - \log(1 - \alpha)}{\log(p_o(1 - p_u)) - \log(p_u(1 - p_o))}$$

$$b_r = \frac{\log(1 - \beta) - \log(\alpha)}{\log(p_o(1 - p_u)) - \log(p_u(1 - p_o))}$$

The maximum number of errors for accepting an inventory after n drawn values is:

$$y_{an} = m \cdot n + b_a$$

Similarly, the minimum number of errors for a rejection of an inventory after n drawn values is:

$$y_{rn} = m \cdot n + b_r$$

n_i defines the number of drawn elements and f_i the number of errors after the i -th drawing. It follows that $n_0 = 0 = f_0$. The number for the next drawing is then calculated by:

$$n_i = \frac{f_{i-1} - b_a}{m}$$

Thus, in the i -th drawing $n_i - n_{i-1}$ new samples need to be drawn. If then $f_i \leq y_{an_i}$ is true, the inventory will be accepted; if $f_i \geq y_{rn_i}$ is true, the inventory is rejected.

1.8.3 Example on a fictitious warehouse

We choose $p_u = 0,5\% = 0,005$ and $p_o = 1\% = 0,01$, and $\alpha = 5\% = 0,05 = \beta$. It follows that:

$$m = 0,007215558 \dots$$

$$b_a = -4,217276347 \dots$$

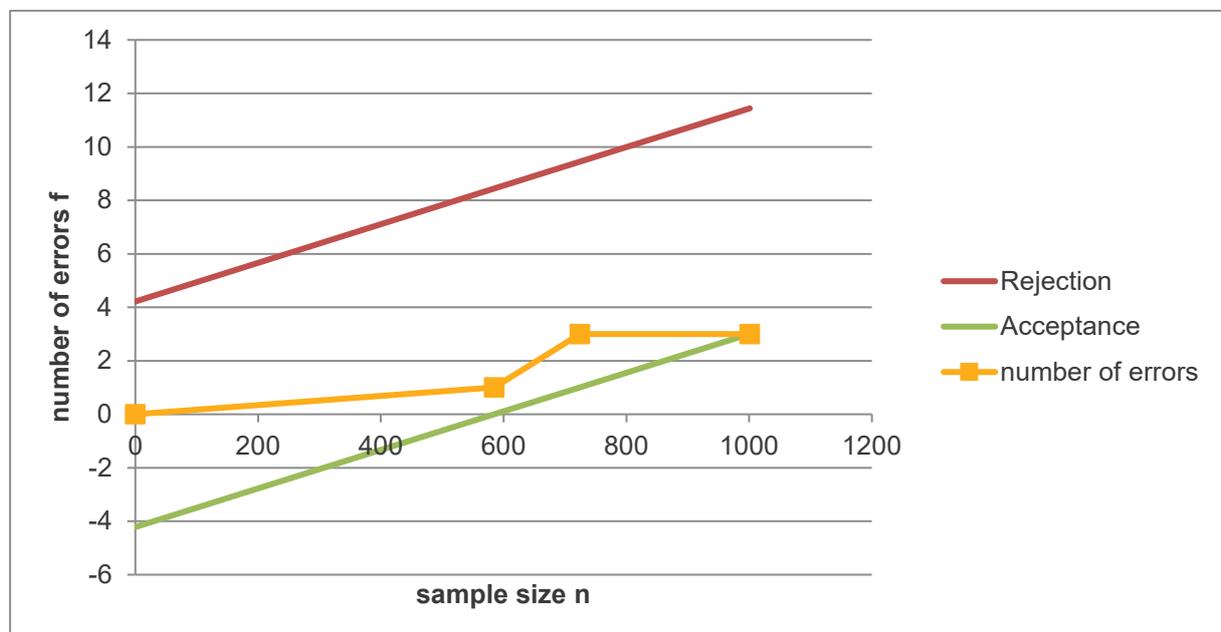
$$b_r = 4,217276347 \dots$$

In this case the equation $V_o = 1 - \beta = 0,95 = 95\% = 1 - \alpha = V_u$ is true.

As $n_0 = 0 = f_0$, we get $n_1 = \frac{-b_a}{m} = 584,4699 \dots \approx 585$. That means we have to draw 585 samples. It follows that $y_{an_1} \geq 0$ and $y_{rn_1} \leq 9$. If the number of errors for these 585 samples is 0, the inventory will be accepted. If the number of errors is 9 or larger, the inventory will be rejected. The sequential test is done in these cases. For all other numbers of errors the drawing needs to be continued. For example (with rounded values):

i	n_i	f_i	y_{an_i}	y_{rn_i}	Action
1	585	1	0	9	Continue drawing
2	724	3	1	10	Continue drawing
3	1001	3	3	12	Acceptance

Attention should be paid to fact that n_i and y_{rn_i} are rounded up and y_{an_i} is rounded down.



1.8.4 Complete enumeration in border area

AWV [AWV, 1985, S. 23] recommends complete enumeration for high values and "for

- Zero inventory/ almost zero inventory,
- Supply differences,

- Innovation,
- High turnover rate and
- Material with high blend

for

- Reasons of anticipated security in order disposition and manufacturing

for

- Precious metals and other valuable materials
- Stocks according to regulations for poison
- Stocks with shrinkage, spoilage and so on and with risk of theft and
- Stocks with high single value (value per customary amount).”

1.9 Glossary of symbols

It was tried, to use symbols following the rules:

- X, Y and N indicate book values, actual values or quantities
- Capital letters refer to the whole, small letters refer to a part
- Index- i and $-h$ indicate the i -th item and the h -th stratum respectively
- Bars, like \bar{y} , indicate mean, partly the mean is specified by an index

General Symbols	
N	Number of elements in basic population
μ	Mean of basic population
σ	Standard deviation
V	Variance
t	Confidence value ("t-value")
Symbols specific to inventory	
Y	Total value of warehouse
d	Acceptable absolute error
Symbols specific to sampling	
n	Number of elements in random sample
y_i	i -th element in sample \hat{y}
\bar{y}	Mean of sample \hat{y}
Symbols specific to stratified sampling	
L	Number of strata
N_h	Number of elements in stratum h
x_h	Maximum value in stratum h
n_h	Number of elements for sampling in stratum h
y_{hi}	i -th element in sample \hat{y}_h
\bar{y}_h	Mean of sample in stratum h
s_h	Standard deviation in stratum h being the square root of the sample variance v
\bar{y}_{st}	Mean of sample \hat{y}_{st} calculated by stratification
Symbols for regression estimation	
R^2	Coefficient of determination
X, x	Auxiliary values (similar to the symbols with Y)
\bar{y}_{lr}	Through linear regression specified mean
B	Proportionality factor between x and y
b	Approximated proportionality factor between x and y
Symbols for sequential testing	
m	Slope of the straight line
b_a	Y intercept with the straight line of acceptance
b_r	Y intercept with the straight line of rejection
y_{an}	Maximum error amount for acceptance after n drawn items
y_{rn}	Minimum error amount for acceptance after n drawn items
p_u	Lower limit of the error portion
p_o	Upper limit of the error portion
V_u	Confidence of the upper limit
V_o	Confidence of the lower limit
α	Risk of the warehouse owner
β	Risk of the inspector
f_i	Number of errors after the i -th drawing
n_i	Number of drawn items after the i -th drawing

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